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Gardon Heat Gage Temperature Response

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Nomenclature

ρ = density, #/ft^3
C_p = specific heat capacity, BTU/#m-°F
k = thermal conductivity, BTU/hr-ft-°F
$\mathscr{T}$ = thickness, in.
$\dot{q}$ = heat flux, BTU/hr-ft^2
T = temperature, °F
r = radius, inner, in.
t = time, sec
T_{ss} = steady state temperature, °F
R = radius, outer, in.
r^* = dimensionless radius ratio
T^* = dimensionless temperature ratio
t^* = dimensionless time ratio
$\bar{T}$ = transformed dimensionless temperature ratio
X = transformed dimensionless temperature ratio
A_j = Bessels coefficients
λ_j = eigenvalues
J_o = Bessels function, zero-order, first kind
J_1 = Bessels function, first-order, first kind
$\bar{T}_N$ = transformed dimensionless temperature ratio evaluated at the new boundary condition
T_{o_s} = initial temperature, °F

Introduction

ONE particular method of directly measuring the heat flux of a convective environment is through the use of a circular foil heat flux (Gardon) gage, whose basic operating principal is that the temperature difference between the center and edge of the foil face is directly proportional to the magnitude of the heat flux energy input to the gage. Inasmuch as the gage is generally calibrated in a controlled radiative environment with its edge temperature held constant, utilization of the gage in an environment where data are taken with a transient edge temperature, should be examined.

Gardon,[1] in originally describing the gage, solved the governing steady-state equation which basically states that incident heat flux is proportional to the foil center and edge temperature difference. The proportionality was found to be linear as a consequence of the fortuitous linear diffusivity of constantan. Gardon also proposed an approximate solution for the case of a uniform edge temperature in an effort to describe the speed of response of the gage. Later, Coffin[2] solved the uniform edge temperature problem for several varying heat flux inputs and addressed the problem of a varying sink temperature by stating that if the edge raises in temperature, the center temperature raises a proportional amount such that the gage output remains constant. An important result of the work of Laganelli and Martellucci[3,4] was that data taken from radiantly calibrated gages used in a convective environment, where test conditions dictated that the data be taken transiently with a rapidly varying edge temperature, agreed very well with data taken on identical models by several other independent and mutually supportive experimental techniques.

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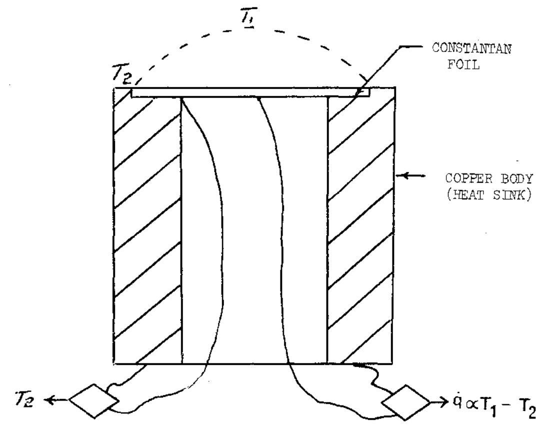

Fig. 1 Schematic of thin foil gage.

Analysis

Figure 1 is a schematic of a typical Gardon thin foil gage. The governing differential equation describing the temperature distribution for the circular foil gage is given by

$$\frac{\rho C_p}{k}\frac{\partial T}{\partial t} = \frac{\dot{q}}{\mathscr{T}k} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \tag{1}$$

which is subject to the boundary conditions (zero edge reference temperature):

$$T(0,t) = \text{finite}; \quad T(0,\infty) = T_{o_s};$$
$$T(R,t) = 0; \quad T(r,0) = 0 \text{ (initially)}$$

The steady-state solution of Eq. (1), is

$$T_{ss} = T_{\text{center}} - T_{\text{edge}} = \dot{q}(R^2/4\mathscr{T}k) \tag{2}$$

That solution shows that heat flux is proportional to the steady-state temperature difference and was first applied to thin-foil heat gages by Gardon.

If the following dimensionless parameters are introduced

$$r = r^*R \tag{3a}$$
$$T = T^*T_{ss} \tag{3b}$$
$$t = t^*R^2/a \tag{3c}$$
$$a = k/\rho C_p \tag{3d}$$

Equation (1) is rewritten as

$$\frac{\rho C_p}{k} + \frac{\partial T^*}{\partial t^*}\frac{T_{ss}}{R^2}\frac{k}{\rho C_p} = \frac{\dot{q}}{\mathscr{T}k} + \frac{1}{r^*}\frac{\partial T^*}{\partial r^*}\frac{T_{ss}}{R} + \frac{\partial^2 T^*}{\partial r^{*2}}\frac{T_{ss}}{R^2} \tag{4}$$

Noting that

$$\dot{q}/\mathscr{T}k = 4(T_{ss}/R^2) \tag{5}$$

Equation (4) simplifies to

$$\frac{\partial T^*}{\partial t^*} = 4 + \frac{1}{r^*}\frac{\partial T^*}{\partial r^*} + \frac{\partial^2 T^*}{\partial r^{*2}} \tag{6}$$

If a linear transformation is applied to T such that

$$T^*(r^*,t^*) = \bar{T}(r^*,t^*) + X(r^*) \tag{7}$$

then the transformed boundary conditions become

$$\bar{T}(r^* = 0, t^*) = \text{finite} \tag{8a}$$
$$\bar{T}(r^* = 0, \infty) = 0 \tag{8b}$$
$$\bar{T}(r^* = 1, t^*) = 0 \tag{8c}$$
$$\bar{T}(r^*, 0) = 0 \tag{8d}$$

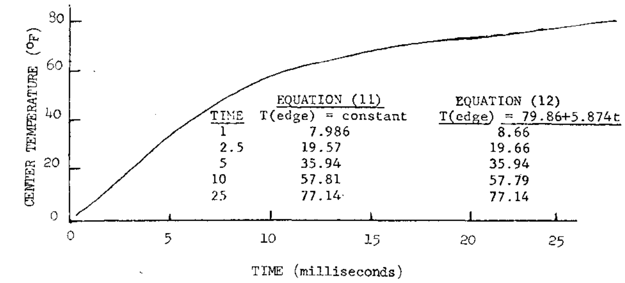

Fig. 2 Equivalence of solutions for constant and varying edge temperatures.

and Eq. (6) becomes

$$\frac{\partial \bar{T}}{\partial t^*} = 4 + \frac{1}{r^*}\frac{\partial \bar{T}}{\partial r^*} + \frac{1}{r^*}\frac{\partial X}{\partial r^*} + \frac{\partial^2 \bar{T}}{\partial r^{*2}} + \frac{\partial^2 X}{\partial r^{*2}} \tag{9}$$

After separating variables and utilizing the transformed boundary conditions, the solution of Eq. (9) becomes

$$T^*(r^*, t^*) = AJ_o(\lambda r^*)\, e^{-\lambda^2 t^*} + 1 - r^{*2} \tag{10}$$

Further application of the boundary conditions, integration, and simplification results in

$$T^*(r^*, t^*) = 2\sum_{j=1}^{\infty} \frac{J_o(\lambda_j r^*)}{[J_1(\lambda_j)]^2}\, e^{-\lambda_j^2 t^*}\left[-\frac{4}{\lambda_j^3} J_1(\lambda_j)\right] + 1 - r^{*2} \tag{11}$$

Equation (11) is the complete solution for the case of uniform edge temperature. For complete details of all mathematical steps, the reader should consult Ref. 5. When the edge temperature becomes a function of time however, with Eq. (9) remaining the same, $T(r = R, t) = f(t)$, the new original boundary condition, is transformed to $T^*(r^* = 1, t^*) = \bar{T}(r^* = 1, t^*)$, with the other transformed boundary conditions remaining the same. By Duhamel's theorem, which basically allows the transient solution to be expressed in terms of the steady-state solution as defined by Eq. (11), it is possible to express the exponential portion of Eq. (11), the only portion that changes, as follows:

$$\bar{T}_N(r^*, t^*) = F(+0)\bar{T}(r^*, t^*) + \int_o^t F'(t-\tau)\bar{T}(r^*, t^*)\, d\tau \tag{12}$$

$F(+0)$ is the edge temperature function evaluated at time $= 0$ while $F'(t-\tau)$ is the first derivative of that same function. The integration thus required is with respect to the time variable and is, generally, of the form

$$\int_o^{t^*} e^{-\lambda_j^2 t^*}(B + 2Ct^* + 3Dt^{*2} + \ldots)\, dt^* \tag{13}$$

where a polynomial edge temperature function has been assumed.

Thus Duhamel's theorem provides a very useful tool with which to solve the case of varying edge temperature once the solution for a uniform edge reference temperature has been established. The following curve provides a numerical example, in lieu of a classical mathematical proof, that the two solutions are equivalent. The edge temperature function, based on data by Laganelli,[3] is given by $f(t) = 79.86 + 5.874t$.

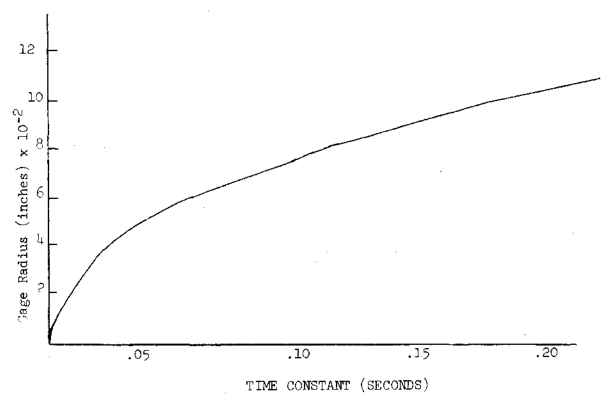

Fig. 3 Gage response capability.

Conclusions

The result shows that even though the edge temperature varies with time, the difference between the center and edge temperature remains constant; thus the gage correctly measures heat flux input through both transient and steady-state sink temperatures. For small values of time, the exponential time function dominates the solution, and it is important to carry more terms in the Bessel summation. As time increases, however, the exponential quickly fades as does the contribution of each succeeding Bessel term. Unlike Fourier summations, only five terms of the Bessel series, with the number decreasing with increasing time, gave very good accuracy. Small gages are controlled by the transient part of the eigenvalues (λ_j's), while the Bessel portion, as defined by the same eigenvalues, controls the geometry. Accuracy, then, does not suffer as the sink temperature varies with time, but does depend on a gage response at least as fast as the edge temperature is changing. The following curve shows time constant vs gage radius, time constant being defined as that time at which temperature reaches 63% of its steady-state value.

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Minimum Heat Transfer Limit in Simple and Gas-Loaded Heat Pipes

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Introduction

THE operational limits of heat pipes in terms of maximum heat flux have been studied by many investigators.[1–4] Some limits have their origins in the vapor flow behavior such as the sonic and the viscous limits,[3,4] while others such as the boiling, capillary, and entrainment limits are not directly related to vapor flow. The minimum heat transfer limit discussed here is mainly a result of the vapor flow phenomenon. It is defined as the lowest rate of heat transfer through the heat pipe below which there no longer exists a section with a uniform temperature higher than the

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